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THE USE OF KNOWN CLASSICAL SYSTEM RELIABILITY ESTIMATION METHODS TO APPROXIMATE THE FINAL SOLUTION IN BAYESIAN METHODOLOGY

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Monterey, California

THESIS

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RELIABILITY ESTIMATION METHODS TO
APPROXIMATE THE FINAL SOLUTION
IN BAYESIAN METHODOLOGY

by

David William Mattis

Thesis Advisor: W. M. Woods

September 1972

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The Use of Known Classical System
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Approximate the Final Solution
in Bayesian Methodology

by

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Lieutenant, United States Navy
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September 1972

ABSTRACT

This thesis examines three methods for calculating the $100(1-\alpha)\%$ lower confidence limits for the reliability of a K-sized series system. Assuming that each component reliability has a Beta distribution, identical posterior parameters A and B are assigned for each component. A computer simulation model is then developed to determine values of $\hat{R}_{s,L(\alpha)}$. The posterior parameters are then converted to attributes data, and $\hat{R}_{s,L(\alpha)}$ is computed using classical CHI-SQUARE methods, and the WOODS-BORSTING Method. The three values are then compared. Although no alternative approximations are examined, the results indicate that a high degree of accuracy in computing $\hat{R}_{s,L(\alpha)}$ is possible with the classical or WOODS-BORSTING methods, and that it may not be necessary to resort to costly simulation techniques to obtain values of $\hat{R}_{s,L(\alpha)}$.

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I. INTRODUCTION

The problem of determining lower confidence limits on the reliability of a K-sized series system using component attributes data only has been studied from the standpoint of both classical methods and Bayesian methods. One classical method, the WOODS-BORSTING Method[1], is used in cases where unequal sample sizes of each component in the series system are tested.

The classical CHI-SQUARE method treats system reliability as a constant, and computes the lower confidence limit by solving the equation

$$\hat{R}_{s,L(\alpha)} = 1 - \frac{\chi^2_{\alpha,2F+2}}{2N}$$

where $\hat{R}_{s,L(\alpha)}$ is the $100(1-\alpha)\%$ lower confidence limit, F is the total number of failures, and N is the common sample size, with each component having an equal sample size for testing. Attributes data, obtained by mission testing, is used to calculate F.

The classical WOODS-BORSTING (W-B) Method computes the lower confidence limit for system reliability from the formula

$$\hat{R}_{s,L(\alpha)} = \exp\left\{-\hat{S}\left[2\hat{r}\right] / \chi^2_{1-\alpha,\left[2\hat{r}\right]}\right\}$$

---

[1]Naval Postgraduate School Report, Special Project No. SP-114, <u>A Method for Computing Lower Reliability Limits Using Component Failure Data with Unequal Sample Sizes</u>, by J. R. Borsting and W. M. Woods, pp. 2-10, June 1968. Hereafter cited as <u>WOODS-BORSTING</u>.

where R_i is the reliability of component i, i=1,2,...,K, and $\hat{S}$ is a logarithmic function of $\prod_{i=1}^{K} R_i$. The term $\hat{r}$ is a function of f_i (the number of failure of component i), and n_i (the sample size of component i).

In the Bayesian methodology, the reliability of each of the K system components is considered to be a random variable. Assuming that component reliability is Beta distributed, a prior set of parameters, A_i and B_i, is assumed. N items of each component are mission tested (ie., tested for a length of time equal to the required mission length), and successes (s_i) and failures (f_i) are obtained for each component i, i = 1,2,...,K. Application of these attributes data to the prior distribution results in the reliability of each component being Beta distributed, with posterior parameters $A_i + s_i$, and $B_i + f_i$. Assuming independence among components, the theoretical system reliability is

$$R_s = \prod_{i=1}^{k} R_i$$

The distribution of R_s is then determined, and the value of $\hat{R}_{s,L(\alpha)}$ is the α^{th} percentile of this posterior distribution.

With systems becoming more costly, the Bayesian approach, with its desirable feature of a smaller required sample size, is becoming more popular. Past mission test data of technically similar systems can be utilized by engineers and scientists to arrive at a prior distribution of the underlying component reliability. The posterior distribution is, however, difficult to determine. The product of individual reliabilities, each Beta distributed, presents formidable

problems to the statistician, with regard to type of distribution, and mathematical tractability.

A computer simulation technique, using randomly generated Beta variables with the desired posterior parameter values can be used to determine the $100(1-\alpha)\%$ lower confidence limit for system reliability. Alternatively, one might approximate this value by converting the posterior parameters into "equivalent" attributes data, and using the $100(1-\alpha)\%$ classical lower confidence limit calculated directly from the formulas given above as the Bayesian lower confidence limit. That is, use known classical system reliability estimation methods to approximate the final solution in Bayesian methodology. The accuracy of this approximation is the basis for the research detailed in this thesis.

II. EXPERIMENTAL PROCEDURE

In order to develop a source of data with which meaningful comparisons could be made, it was necessary to make some basic simplifying assumptions. It was decided that an assumed posterior distribution for each component would be utilized. Further, it was decided that each component in the K-sized series system would have identical posterior parameters. As a result, the parameters A_i and B_i become simply A and B.

The next step was to develop a computer simulation which could be used to generate the distribution of system reliability, utilizing the given Beta parameters A and B. From this distribution, the value of $\hat{R}_{s,L(\alpha)}$ could then be determined.

Next, the posterior parameters would be converted into attributes data. The classical formula and the WOODS-BORSTING formula could then be applied and corresponding values of system reliability lower confidence limits determined.

After tabulating these values for $\hat{R}_{s,L(\alpha)}$, obtained by means of the three methods, according to A and B values, as well as by K, the system size, an attempt would be made to identify which tended to give "more accurate" results. In order to assist in this identification, three other

values would be calculated. These were $\hat{R}^K$, $1-(1-\hat{R})K$, and F (total failures), where $\hat{R}$ is the estimated mean of the posterior distribution, defined by $\hat{R} = \frac{A}{A+B}$.

A. COMPUTER SIMULATION

Let X be a random variable with an exponential distribution. Let

$$-\ln R = \sum_{i=1}^{B} X_i$$

where

$$X_1 \text{ is } \exp(A)$$

$$X_2 \text{ is } \exp\ (A+1)$$

$$\vdots$$

$$X_B \text{ is } \exp(A+B-1)$$

Then R has a Beta distribution, and[2]

$$R = \exp(-\sum_{i=1}^{B} X_i)$$

A computer program was developed to generate a series of B random exponential numbers.[3] Using the above theorem these numbers were then summed and converted to a single random variable, with a Beta distribution, and parameters A and B.

---

[2] Operations Evaluation Group, Center for Naval Analyses of the University of Rochester, Research Contribution No. 79, Confidence Limits for System Reliability, by J. Bram, Appendix A, 15 March 1968.

[3] Emshoff, James R. and Sisson, Roger L., Design and Use of Computer simulation Models, p. 179, McMillan, 1970.

This procedure was then repeated K times, storing the numbers, and multiplying the numbers together as they were developed. The result was a single number for system reliability. The complete procedure was then repeated 1000 times, storing each value of system reliability. The 1000 values were then placed in order from low to high. This, then, was the distribution of system reliability, based upon the given values for A and B. $\hat{R}_{s,L(\alpha)}$ was then the α^{th} percentile of this distribution (ie., α =.10, then R(100) is the desired estimate). This program was used to generate values of $\hat{R}_{s,L(\alpha)}$ over a range of values for A, B, and K.

B. WOODS-BORSTING FORMULA

This formula provides a method for computing lower confidence limits on the reliability of a series system. The formula is repeated below:[4]

$$\hat{R}_{s,L(\alpha)} = \exp\left\{-\hat{S}\left[2\hat{r}\right] / \chi^2_{1-\alpha,[2\hat{r}]}\right\}$$

where

$$\hat{S} = \sum_{i=1}^{K}\left[\frac{2n_i-3}{2(n_i-1)}\hat{Q}_i + \frac{n_i}{n_i-1}\frac{\hat{Q}_i^2}{2}\right], \qquad \hat{Q}_i = \frac{f_i}{n_i}$$

and

$$\hat{r} = \left\{\sum_{i=1}^{K}\left[\frac{2n_i-3}{2(n_i-1)}\hat{Q}_i + \frac{n_i}{n_i-1}\frac{\hat{Q}_i^2}{2}\right]\right\}^2 \Bigg/ \sum_{i=1}^{K}\left[\frac{2n_i-3}{2(n_i-1)}\frac{\hat{Q}_i}{n_i} + \frac{\hat{Q}_i^2}{n_i-1}\right]$$

---

[4] <u>WOODS-BORSTING</u>, pp. 2-8.

with a continuity correction to improve accuracy.

The accuracy of the above formula is a function of the amount of testing, relative to component unreliability[5]

$$F = n_i Q_i$$

A value of F in excess of 5 is considered to be sufficient for reasonable accuracy of $\hat{R}_{s,L(\alpha)}$.

C. CHI-SQUARE FORMULA

For a series system with independent components, the system reliability is the product of the component reliabilities.

$$R_s = \prod_{i=1}^{K} R_i$$

To find the 100(1-α)% lower confidence limit $\hat{R}_{s,L(\alpha)}$ for R_s, one proceeds as follows:

Using mission test data only on component parts, with equal sample sizes, the values of n_i (sample size), f_i (number of failures), and s_i (number of successes) for component i, i=1,2,...,K are obtained.

Then

$$\hat{R}_i = \frac{s_i}{n_i}$$

and

$$\hat{R}_s = \prod_{i=1}^{K} \hat{R}_i$$

---

[5] WOODS-BORSTING, p. 9.

Rewriting gives

$$R_s = \prod_{i=1}^{K}(1-\hat{Q}_i) \;, \qquad \hat{Q}_i = \frac{f_i}{n_i}$$

If $\hat{Q}_i$ is small this can be written as

$$\hat{R}_s = \prod_{i=1}^{K}(1-\hat{Q}_i) = 1 - \sum_{i=1}^{K}\hat{Q}_i$$

The term n_iQ_i is distributed approximately[6] Poisson(n_iQ_i) for $Q_i < .1$, and $n_iQ_i > 5$. Therefore

$$\sum_{i=1}^{K} n_iQ_i = \sum_{i=1}^{K} f_i \dot{\sim} P\left(\sum_{i=1}^{K} n_iQ_i\right) \dot{\sim} P\left(n\sum_{i=1}^{K} Q_i\right)$$

for equal sample sizes $n_1=n_2=\ldots=n_k=n$.

If X is distributed Poisson (λ) and there is one observation on X, then[7]

$$\hat{\lambda}_{U(\alpha)} = \frac{\chi^2_{\alpha,2X+2}}{2}$$

and since $\sum_{i=1}^{K} f_i$ is approximately Poisson $(n\sum_{i=1}^{K} Q_i)$, then

$$\widehat{\left[n\sum_{i=1}^{K} Q_i\right]}_{U(\alpha)} = \frac{\chi^2_{\alpha,2F+2}}{2} \;, \quad F = \sum_{i=1}^{K} f_i$$

and

$$\widehat{\left[\sum_{i=1}^{K} Q_i\right]}_{U(\alpha)} = \frac{\chi^2_{\alpha,2F+2}}{2n} = 1-\hat{R}_{s,L(\alpha)}$$

Therefore

$$\hat{R}_{s,L(\alpha)} = 1 - \frac{\chi^2_{\alpha,2F+2}}{2n}$$

---

[6] Parzen E., <u>Modern Probability Theory and Its Applications</u>, p. 245, 404, Wiley, 1960.

[7] Lloyd, D. K. and Lipow, M., <u>Reliability: Management, Methods, and Mathematics</u>, p. 218, Prentice-Hall, 1962.

This formula is accurate if the underlying assumptions used in its derivation are valid, that is, Q_i is "small," and $n_i Q_i$ greater than 5. A measure of Q_i being small enough can be developed from the maximum likelihood estimators of R_i and R_s.

Recall

$$R_s = \prod_{i=1}^{K} R_i$$

then

$$\hat{R}_s = \prod_{i=1}^{K} \hat{R}_i = 1 - \sum_{i=1}^{K} (1-\hat{R}_i) \quad \text{for small } Q_i$$

and

$$(\hat{R})^K = 1 - K(1-\hat{R}) \quad \text{for identical attributes data}$$

If these two values are in close agreement with each other, then one may assume that $\hat{R}_{s,L(\alpha)}$, calculated from the classical CHI-SQUARE formula, is an accurate estimate. $\hat{R}$, a point estimate for R, is the mean of the Beta distribution; that is

$$\hat{R} = \frac{A}{A+B}$$

D. OBTAINING ATTRIBUTES DATA

In the application of Bayesian statistics with a given prior Beta distribution of R, parameters A' and B' are assumed, based on some measure of how reliable a component is. For example, if no knowledge of the component reliability is available, a uniform prior (A'=B'=1) is assigned. This tends to be the exception rather than the rule, since it is equivalent in the classical sense of allowing only one successful test before actual mission testing takes place.

In essence, the assignment of prior probabilities A' and B' is equivalent in the classical sense of allowing A'+B'-1 tests, with A' successes, prior to actual mission testing. This is true because the α^{th} percentile point of the Beta distribution is the lower $100(1-\alpha)\%$ confidence limit for R when there are A'+B'-1 Bernoulli trials, each with probability of success R, and A' of these trials are successful.[8] After mission testing N items, with S successes and F failures, the posterior distribution of component reliability is Beta distributed, with parameters A=A'+S, and B=B'+F. This in turn is equivalent to having had

$$(A'+S)+(B'+F)-1$$

$$= \quad A+B-1 \quad \text{tests,}$$

with

$$A'+S$$

$$=A \qquad \text{successes}$$

and

$$(B'+S)-1$$

$$=B-1 \qquad \text{failures}$$

Thus, given posterior parameters A and B, the conversion to attributes data is

Number of tests	A+B-1
Successes	A
Failures	B-1

---

[8] DeGroot, Morris H., Optimal Statistical Decisions, p. 160, McGraw-Hill, 1970.

III. PRESENTATION OF DATA

Values of $\hat{R}_{s,L(\alpha)}$ obtained by means of the simulation (SIM) model, the WOODS-BORSTING (W-B) Method, and the CHI-SQUARE or classical (CL) method are presented on the following pages. The values of A, B, and K used were:

A; 50, 100, 150

B; 2, 3, 4

K; 3, 5, 7, 9, 10, 20, 30, 40

In addition, α values of .05, .10 and .20 were used.

To use the tables, first find the page with the desired values of A and B. Read down the left column until the desired value of K is located. The values of $\hat{R}_{s,L(\alpha)}$ are arranged to the right, under the indicated α value.

The point estimate, $\hat{R}$, for each component are given at the top of each table. In addition, the right side of each table lists values of F (total failures), as well as $\hat{R}^k$ and $1-(1-\hat{R})K$.

As an example of how to use the tables, suppose one wishes to compare values of $\hat{R}_{s,L(.10)}$, for A=100, B=3, and K=7. Turning to the correct table and reading down the left and across, one finds

A=100 B=3 $\hat{R}$=.971

		α =.10			
K	. . .	SIM	CL	W-B	. . .
.					.
.					.
.					.
7	. . .	.76	.80	.81	. . .

A=50 B=2 $\hat{R}$=.961

	α =.05			α =.10			α =.20					
K	SIM	CL	W-B	SIM	CL	W-B	SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.81	.85	.80	.83	.87	.84	.86	.89	.88	3	.889	.885
5	.74	.80	.77	.76	.82	.81	.78	.84	.84	5	.822	.808
7	.67	.74	.74	.69	.77	.77	.72	.80	.81	7	.760	.731
9	.61	.69	.70	.63	.72	.74	.65	.75	.77	9	.703	.654
10	.58	.67	.69	.60	.70	.72	.63	.73	.76	10	.676	.615
20	.37	.43	.54	.39	.47	.58	.41	.51	.61	20	.456	.231
30	.24	.21	.44	.25	.25	.46	.27	.30	.49	30	.308	.000
40	.15	.00	.35	.16	.03	.37	.18	.09	.40	40	.208	.000

A=50 B=3 $\hat{R}$=.943

	α=.05			α=.10			α=.20					
K	SIM	CL	W-B	SIM	CL	W-B	SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.76	.77	.75	.78	.80	.79	.80	.83	.82	6	.840	.830
5	.66	.68	.69	.68	.70	.72	.70	.74	.76	10	.747	.717
7	.56	.58	.63	.59	.61	.66	.62	.65	.70	14	.665	.604
9	.49	.49	.57	.52	.52	.60	.54	.57	.64	18	.592	.491
10	.46	.44	.55	.48	.48	.58	.51	.52	.61	20	.558	.434
20	.24	.00	.35	.25	.05	.38	.27	.11	.40	40	.312	.000
30	.12	.00	.23	.13	.00	.25	.15	.00	.27	60	.174	.000
40	.07	.00	.15	.07	.00	.16	.08	.00	.18	80	.097	.000

A=50 B=4 $\hat{R}$=.926

	α =.05			α =.10			α =.20					
K	SIM	CL	W-B	SIM	CL	W-B	SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.70	.70	.70	.73	.73	.74	.75	.76	.77	9	.794	.778
5	.58	.57	.62	.61	.60	.65	.63	.64	.68	15	.681	.630
7	.49	.43	.54	.51	.47	.57	.53	.51	.60	21	.584	.482
9	.41	.30	.47	.43	.34	.50	.46	.39	.53	27	.500	.333
10	.37	.24	.44	.40	.28	.47	.42	.33	.50	30	.463	.259
20	.16	.00	.23	.17	.00	.25	.18	.00	.27	60	.214	.000
30	.07	.00	.12	.07	.00	.13	.08	.00	.15	90	.099	.000
40	.03	.00	.07	.03	.00	.07	.04	.00	.08	120	.046	.000

A=100 B=2 $\hat{R}$=.980

K	α=.05 SIM	α=.05 CL	α=.05 W-B	α=.10 SIM	α=.10 CL	α=.10 W-B	α=.20 SIM	α=.20 CL	α=.20 W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.90	.92	.89	.91	.93	.92	.93	.95	.94	3	.942	.941
5	.86	.90	.88	.87	.91	.90	.88	.92	.92	5	.906	.902
7	.82	.87	.86	.83	.88	.88	.84	.90	.90	7	.871	.863
9	.78	.84	.84	.79	.86	.86	.81	.88	.88	9	.837	.824
10	.76	.83	.82	.77	.85	.85	.79	.86	.87	10	.820	.804
20	.60	.71	.74	.62	.73	.76	.64	.75	.78	20	.673	.608
30	.48	.60	.66	.50	.62	.68	.52	.65	.70	30	.552	.412
40	.39	.49	.59	.40	.51	.61	.42	.54	.63	40	.453	.216

$A=100 \quad B=3 \quad \hat{R}=.971$

	$\alpha = .05$			$\alpha = .10$			$\alpha = .20$					
K	SIM	CL	W-B	SIM	CL	W-B	SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.87	.88	.87	.88	.90	.89	.89	.91	.91	6	.915	.913
5	.81	.84	.83	.82	.85	.85	.84	.87	.87	10	.863	.854
7	.75	.79	.79	.76	.80	.81	.78	.82	.83	14	.813	.796
9	.70	.74	.75	.72	.76	.77	.74	.78	.80	18	.766	.738
10	.67	.72	.74	.69	.74	.76	.71	.76	.78	20	.744	.709
20	.49	.49	.59	.50	.52	.61	.52	.55	.63	40	.554	.418
30	.35	.27	.47	.36	.30	.49	.38	.34	.51	60	.412	.126
40	.25	.06	.38	.27	.09	.40	.28	.13	.42	80	.307	.000

A=100 B=4 $\hat{R}=.961$

	$\alpha=.05$			$\alpha=.10$			$\alpha=.20$					
K	SIM	CL	W-B	SIM	CL	W-B	SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})K$
3	.84	.85	.84	.85	.86	.86	.86	.88	.88	3	.889	.885
5	.76	.78	.78	.78	.79	.80	.79	.81	.82	15	.822	.808
7	.69	.71	.73	.71	.73	.75	.73	.75	.77	21	.760	.731
9	.64	.64	.68	.65	.66	.70	.67	.69	.72	27	.702	.654
10	.61	.61	.66	.62	.63	.68	.64	.65	.70	30	.676	.615
20	.39	.28	.48	.41	.31	.49	.42	.34	.51	60	.456	.231
30	.26	.00	.34	.27	.00	.36	.28	.04	.38	90	.308	.000
40	.17	.00	.25	.18	.00	.26	.19	.00	.28	120	.208	.000

A=150 B=2 $\hat{R}$=.987

	α =.05			α =.10			α =.20					
K	SIM	CL	W-B	SIM	CL	W-B	SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.93	.95	.93	.94	.96	.94	.95	.96	.96	3	.961	.961
5	.90	.93	.92	.91	.94	.93	.92	.95	.94	5	.936	.934
7	.87	.91	.90	.88	.92	.92	.89	.93	.93	7	.911	.908
9	.85	.90	.89	.85	.91	.90	.87	.92	.92	9	.888	.882
10	.83	.89	.88	.84	.90	.89	.86	.91	.91	10	.876	.868
20	.72	.81	.81	.73	.82	.83	.74	.84	.85	20	.767	.737
30	.62	.73	.75	.63	.75	.77	.64	.76	.79	30	.672	.605
40	.53	.66	.70	.54	.67	.72	.56	.69	.73	40	.589	.474

A=150 B=3 $\hat{R}$=.980

K	α =.05 SIM	CL	W-B	α =.10 SIM	CL	W-B	α =.20 SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.91	.92	.91	.92	.93	.92	.93	.94	.94	6	.942	.941
5	.87	.89	.88	.88	.90	.89	.89	.91	.91	10	.906	.902
7	.82	.86	.85	.84	.87	.87	.85	.88	.88	14	.871	.863
9	.79	.83	.83	.80	.84	.84	.82	.85	.86	18	.837	.824
10	.77	.81	.81	.78	.82	.83	.80	.84	.85	20	.820	.804
20	.62	.66	.70	.63	.68	.72	.64	.70	.73	40	.673	.608
30	.49	.51	.61	.51	.53	.62	.52	.56	.64	60	.552	.412
40	.40	.37	.52	.41	.39	.54	.42	.42	.56	80	.453	.216

$A=150$ $B=4$ $\hat{R}=.974$

	$\alpha=.05$			$\alpha=.10$			$\alpha=.20$					
K	SIM	CL	W-B	SIM	CL	W-B	SIM	CL	W-B	F	$\hat{R}^k$	$1-(1-\hat{R})k$
3	.89	.90	.89	.90	.91	.90	.91	.92	.92	9	.924	.922
5	.83	.85	.85	.85	.86	.86	.86	.87	.88	15	.877	.870
7	.78	.80	.81	.79	.82	.82	.81	.83	.84	21	.832	.818
9	.74	.76	.77	.75	.77	.79	.76	.79	.81	27	.789	.766
10	.72	.74	.76	.73	.75	.77	.74	.77	.79	30	.769	.740
20	.53	.51	.61	.55	.54	.62	.56	.56	.64	60	.591	.480
30	.40	.30	.49	.41	.32	.50	.43	.35	.52	90	.454	.221
40	.30	.09	.40	.31	.12	.41	.32	.15	.42	120	.349	.000

IV. CONCLUSIONS

In order to present the tabular values of $\hat{R}_{s,L(\alpha)}$ in a more meaningful manner, they were plotted as a dependent variable, using K as the independent parameter, for a fixed value of A, B, and α. Three curves were plotted, one for each method. In addition a new variable Δ was defined as

$$\Delta \equiv \left| \hat{R}^k - \left[1-(1-\hat{R})k\right] \right| \quad , \quad R = \frac{A}{A+B}$$

and was plotted on the same graph, depending on the values of A and B. This was done to show the increase in the error of $\hat{R}_{s,L(\alpha)}$ with K due to the simplifying assumptions made in developing the CHI-SQUARE formula. These curves are shown in APPENDIX A.

For the case A=50, B=2, plots were made for all three α values. There was no major difference observed in the curves as α was varied. As expected, the curves were higher for a given value of K as α was increased. The plots were looked at from the point of view of ranges of $\hat{R}$, as defined by A/A+B. Two cases were examined, $\hat{R} \geq .98$ and $R < .98$.

A. CASE I; $\hat{R} \geq .98$

For values of $\hat{R} \geq .98$ the value of Δ as a function of K increased gradually. Both the W-B and CL curves provided optimistic values of $\hat{R}_{s,L(\alpha)}$, using the simulation curve (SIM) as a basis for comparison. The classical curve gave a smaller relative error except for values of K less than

about 10, where the W-B curve was closer to the SIM curve. All three curves showed the same trend downward as K increased, and all three curves were reasonably close to each other.

B. CASE II; $\hat{R} < .98$

For values of $\hat{R} < .98$ the curve of $\hat{R}_{s,L(\alpha)}$ for the classical method crossed the SIM curve, providing an optimistic prediction for $\hat{R}_{s,L(\alpha)}$ at lower values of K and a pessimistic value for higher K values. This changeover occured for values of K as low as 5 and a high as 35, increasing as $\hat{R}$ increased. The W-B curve was once again consistently optimistic compared with the simulation curve. The Δ curve increased rapidly for $R < .98$.

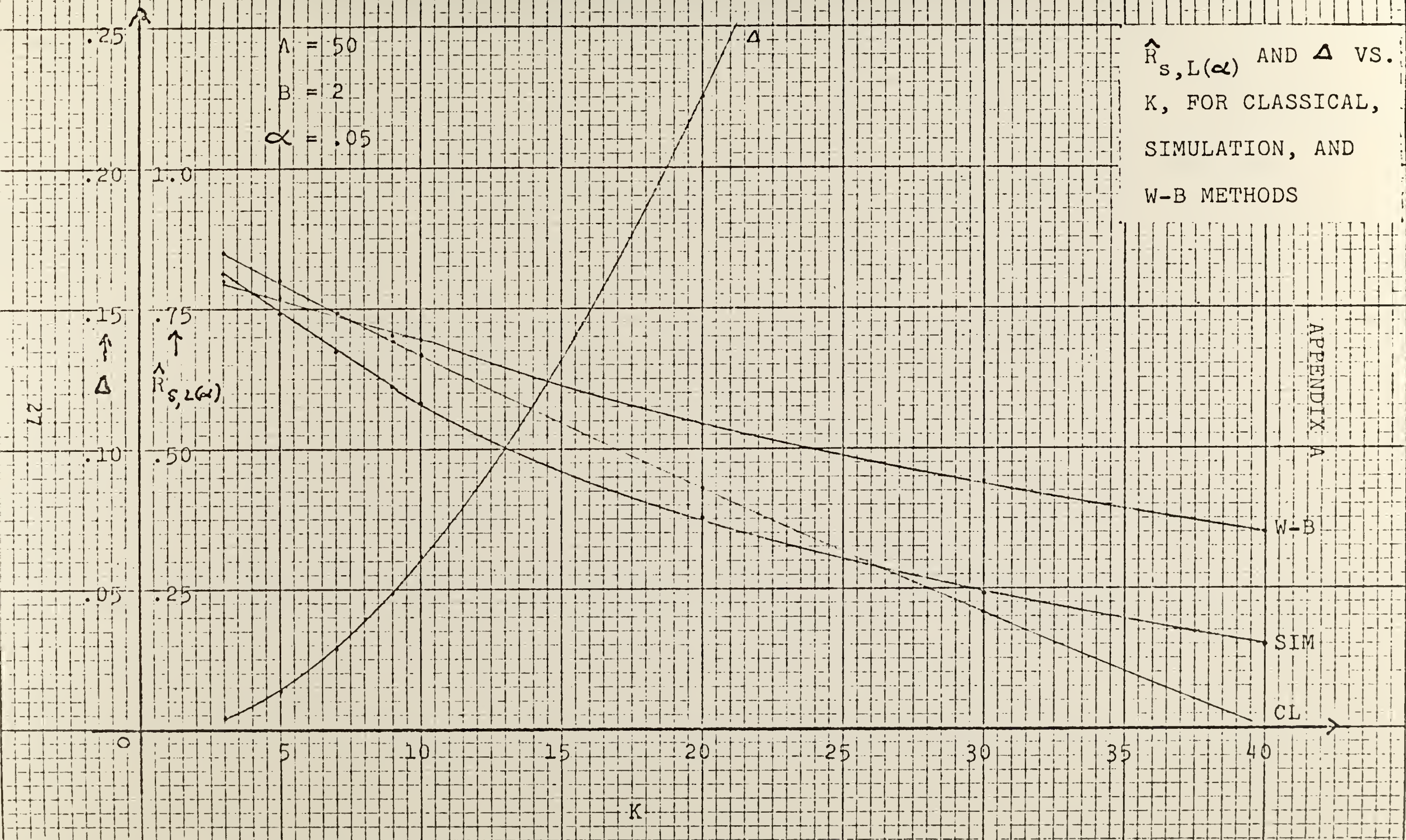
C. GENERAL CONCLUSIONS

Since the system reliabilities which would logically be assumed to be of importance to scientists and engineers are those in excess of .75, especially in view of high system cost considerations, it is concluded from the plots that for a value of $\hat{R}_{s,L(\alpha)}$, as obtained by the simulation, in excess of .75, both the classical and the WOODS-BORSTING Methods give quite good approximations to the final solution obtained in Bayesian methodology. Even though both the WOODS-BORSTING and the classical formulas yield optimistic values, the maximum error in the range of (SIM) $\hat{R}_{s,L(\alpha)} \geq .75$ is .10, for the parameters used.

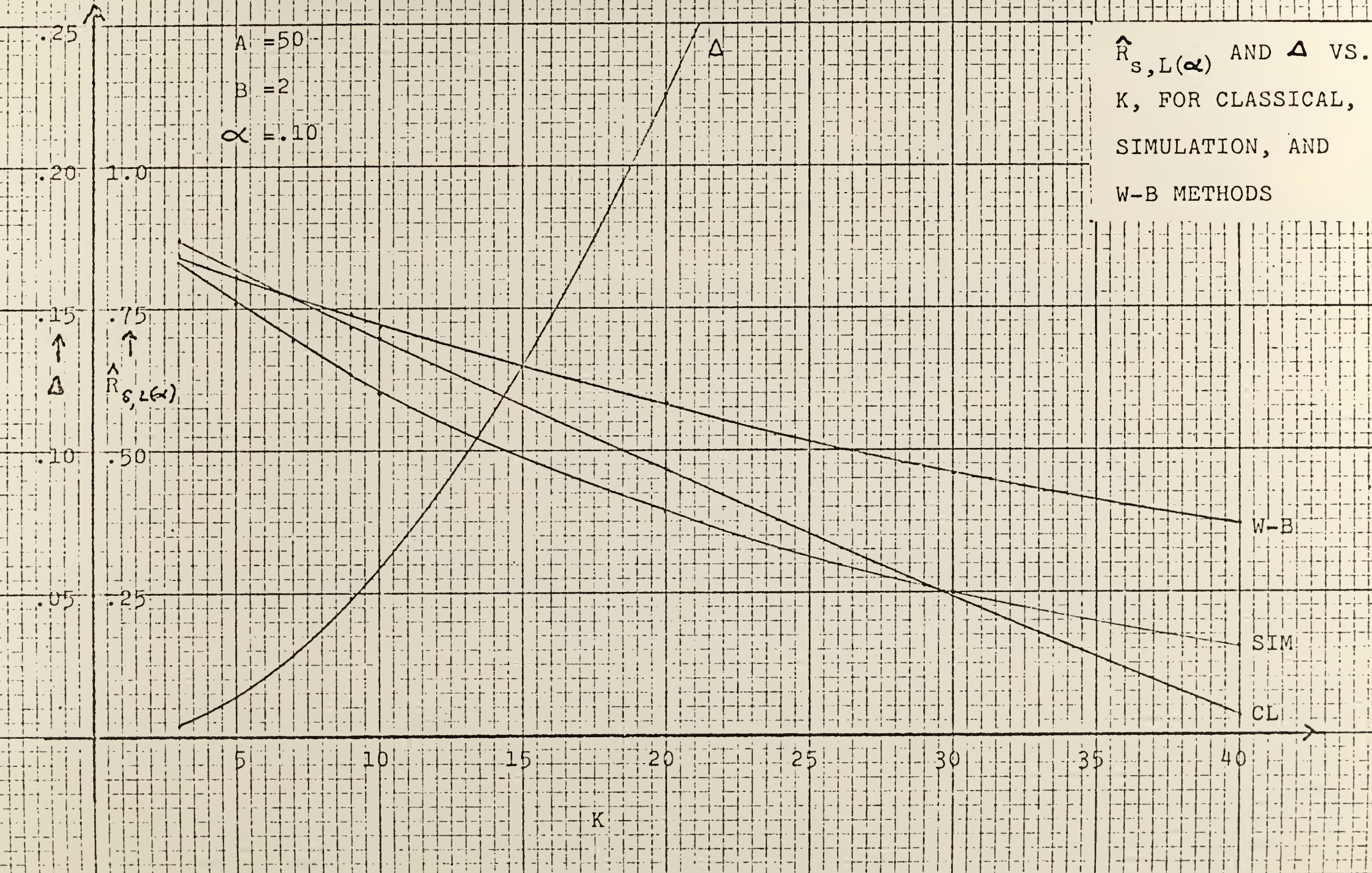
D. AREAS OF FUTURE STUDY

The parameters used were chosen such that each component, when converted to attributes data, had the same sample size as all of the remaining K-1 components. This was done to ascertain whether or not the classical formula, valid only for equal sample sizes among components, could be used as an approximation to the final solution. For unequal sample sizes, the WOODS-BORSTING formula must be used -- the classical formula does not apply. It remains to be seen whether or not the estimates obtained by means of the simulation model can be accurately approximated by the W-B formula, with unequal sample sizes, and varying numbers of failures, for each component. The K value was not examined beyond 40. A trend is evident on the curves, but research should be done for $K > 40$, rather than a mere extrapolation of the data.

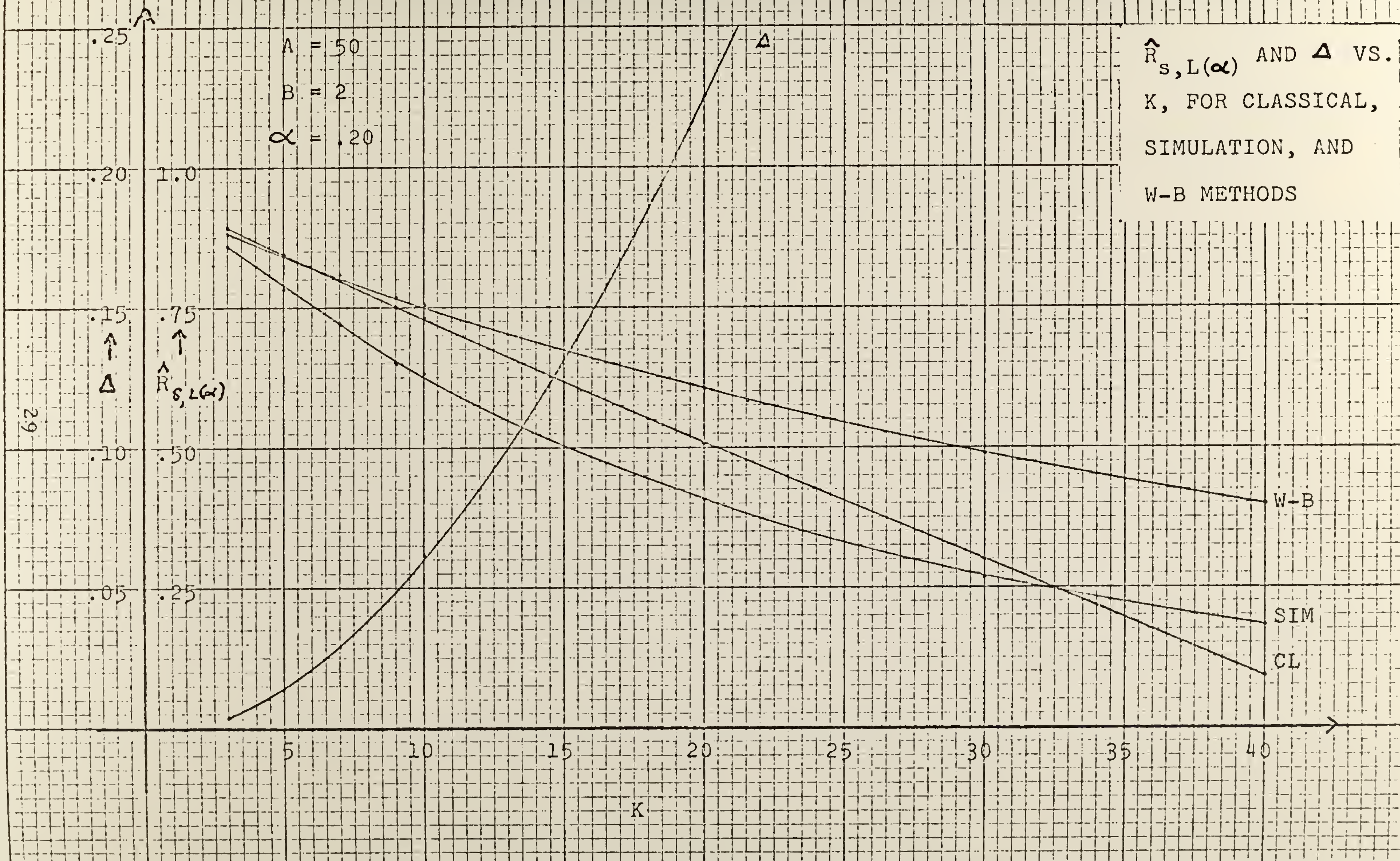
Parameter values studied for A and B were limited to those presented. A posterior parameter for A in excess of 150 is necessary to yield high system reliability confidence limits for values of K in excess of 15. This area was not examined.



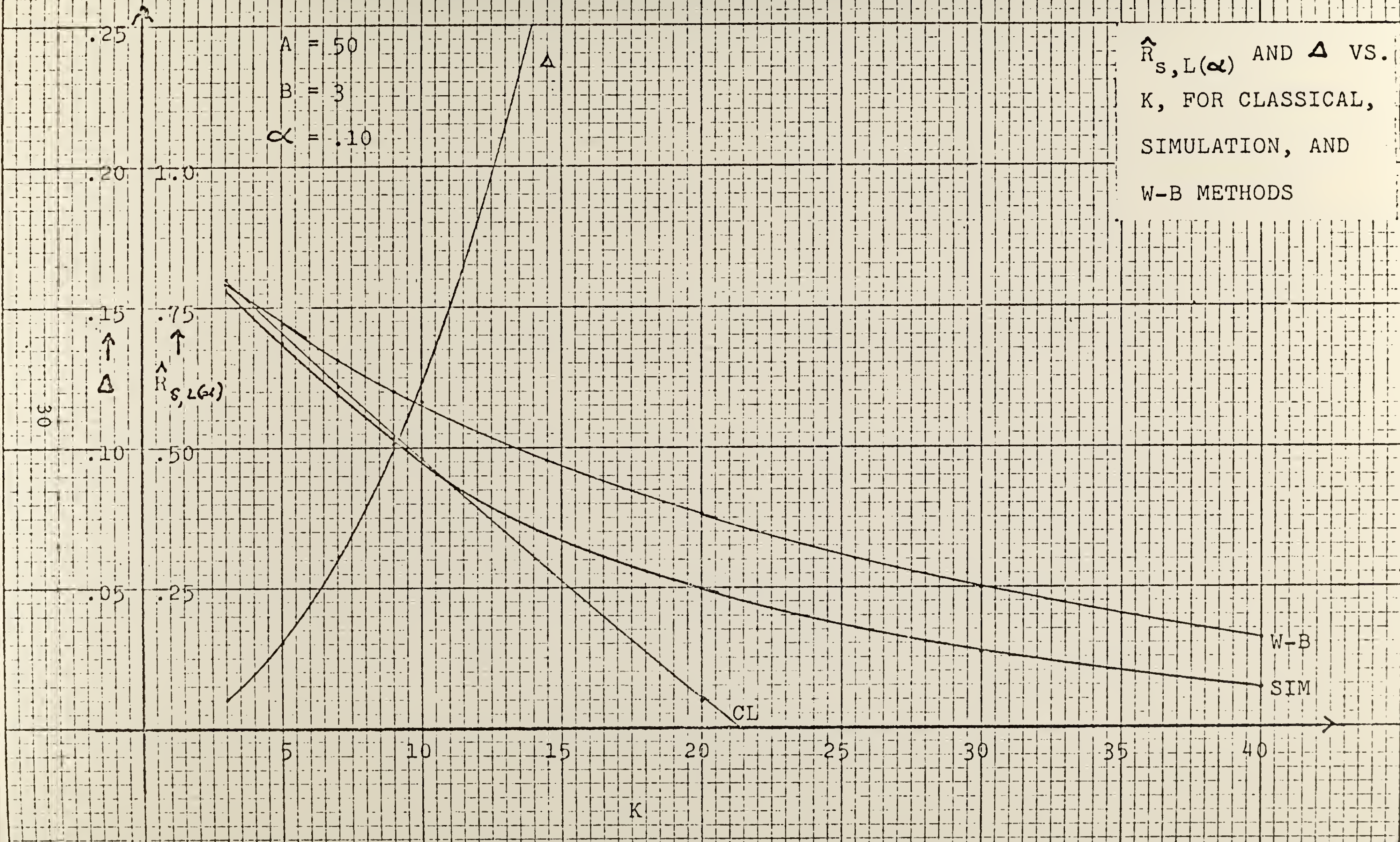
$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS



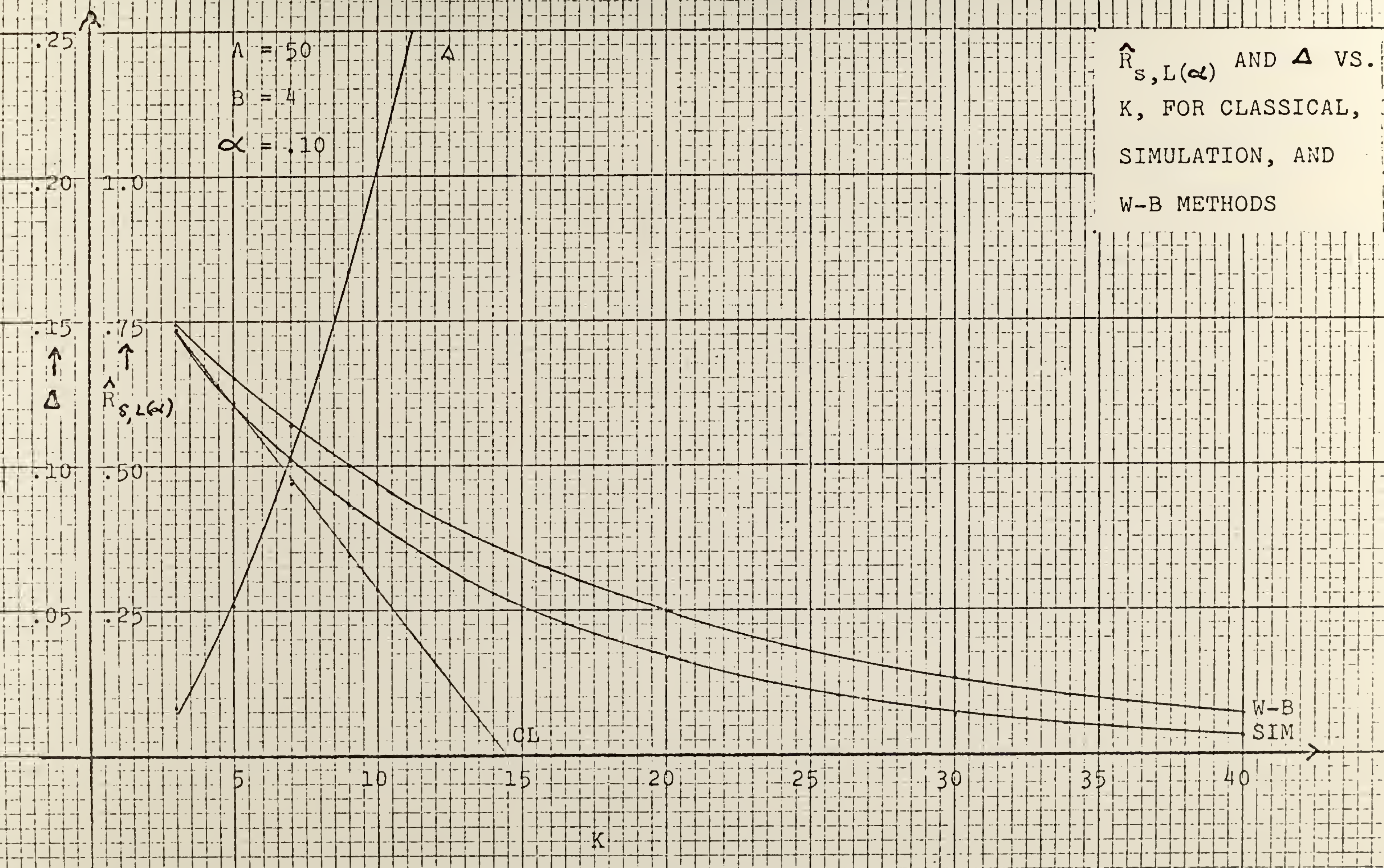
$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS

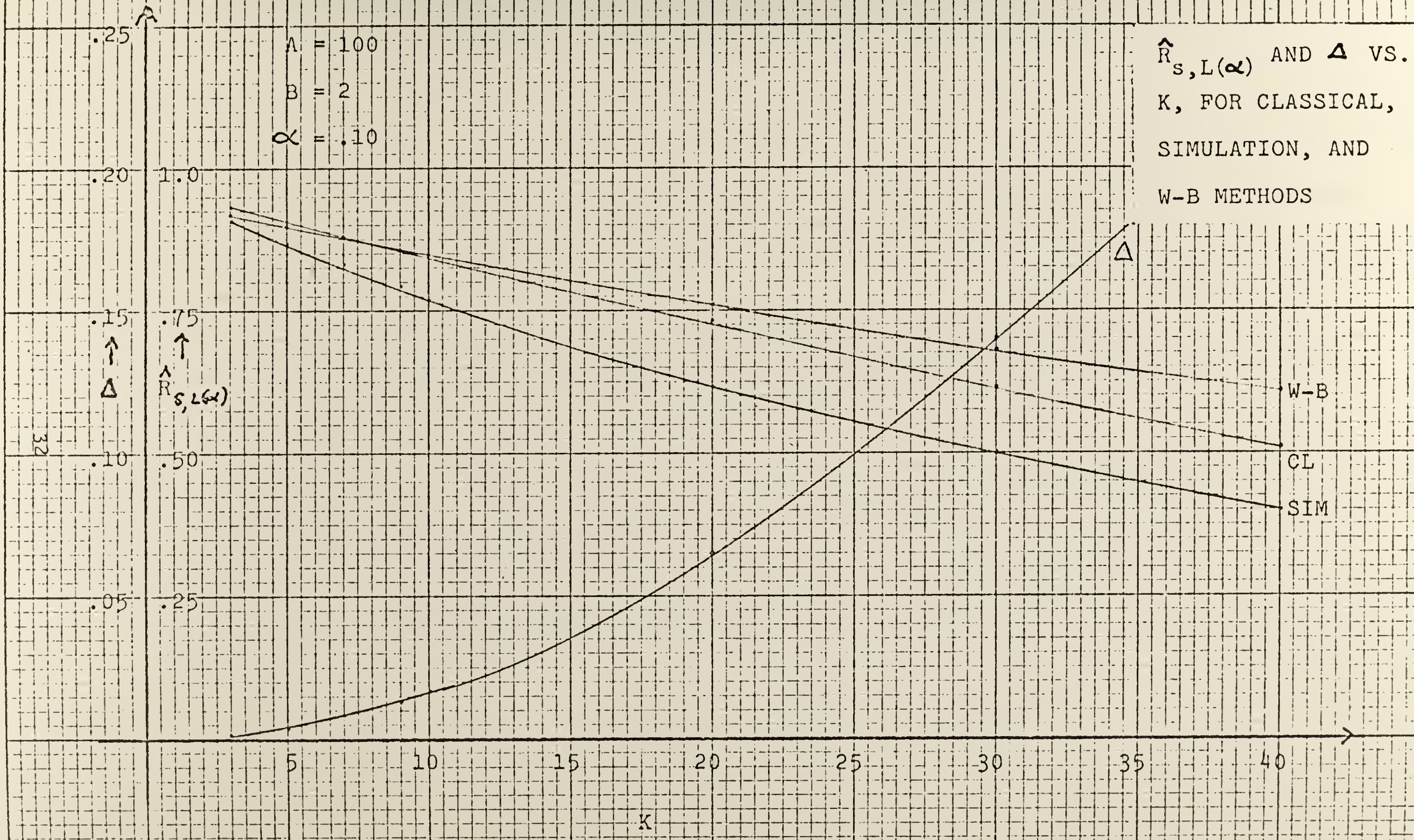


$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS
A = 50
B = 2
α = .20
.25
.20
.15
.10
.05
Δ
1.0
.75
.50
.25
$\hat{R}_{s,L(\alpha)}$
5
10
15
20
25
30
35
40
K
W-B
SIM
CL

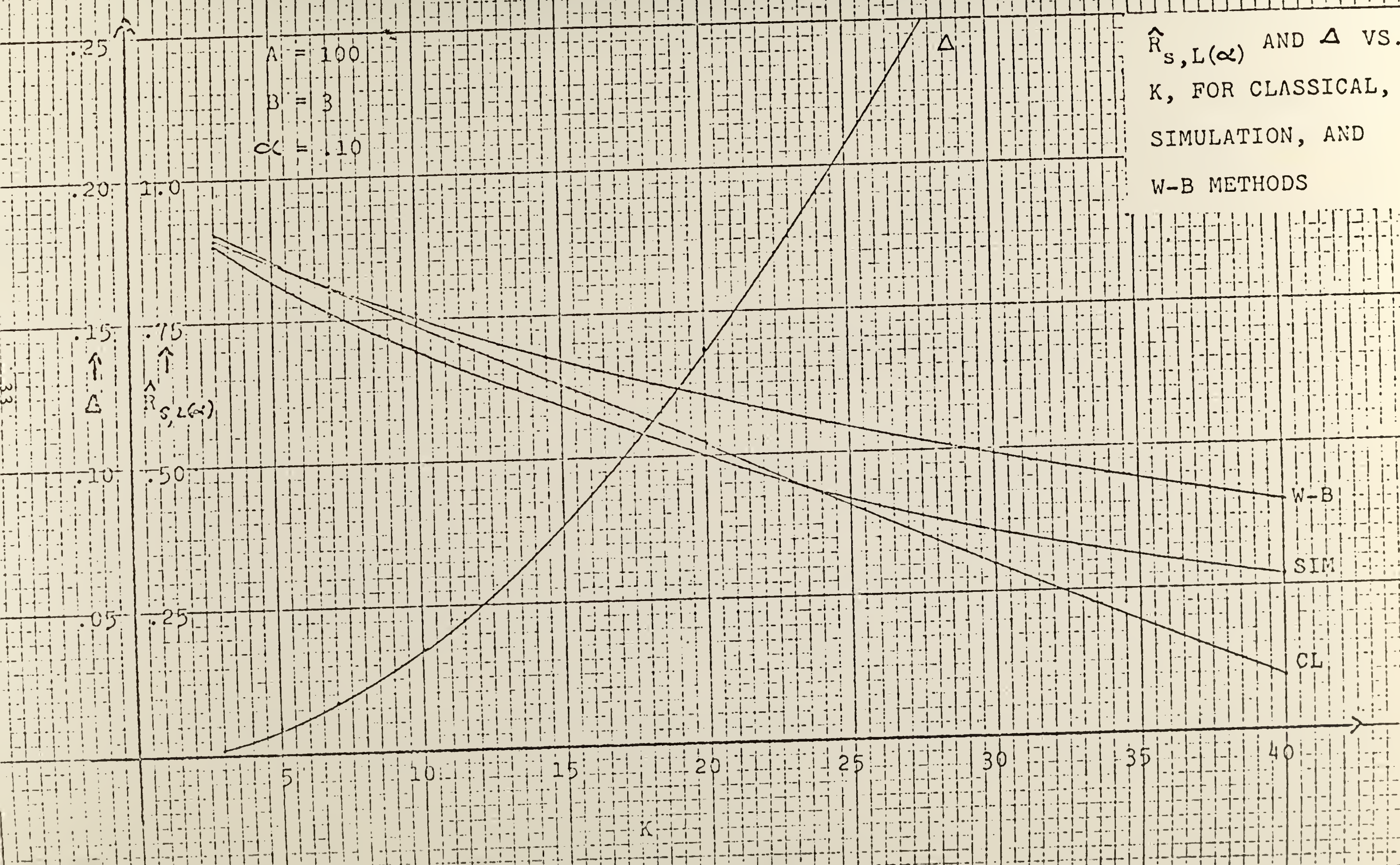


$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS
A = 50
B = 3
α = .10
Δ
$\hat{R}_{s,L(\alpha)}$
.25
.20
.15
.10
.05
1.0
.75
.50
.25
5
10
15
20
25
30
35
40
K
W-B
SIM
CL

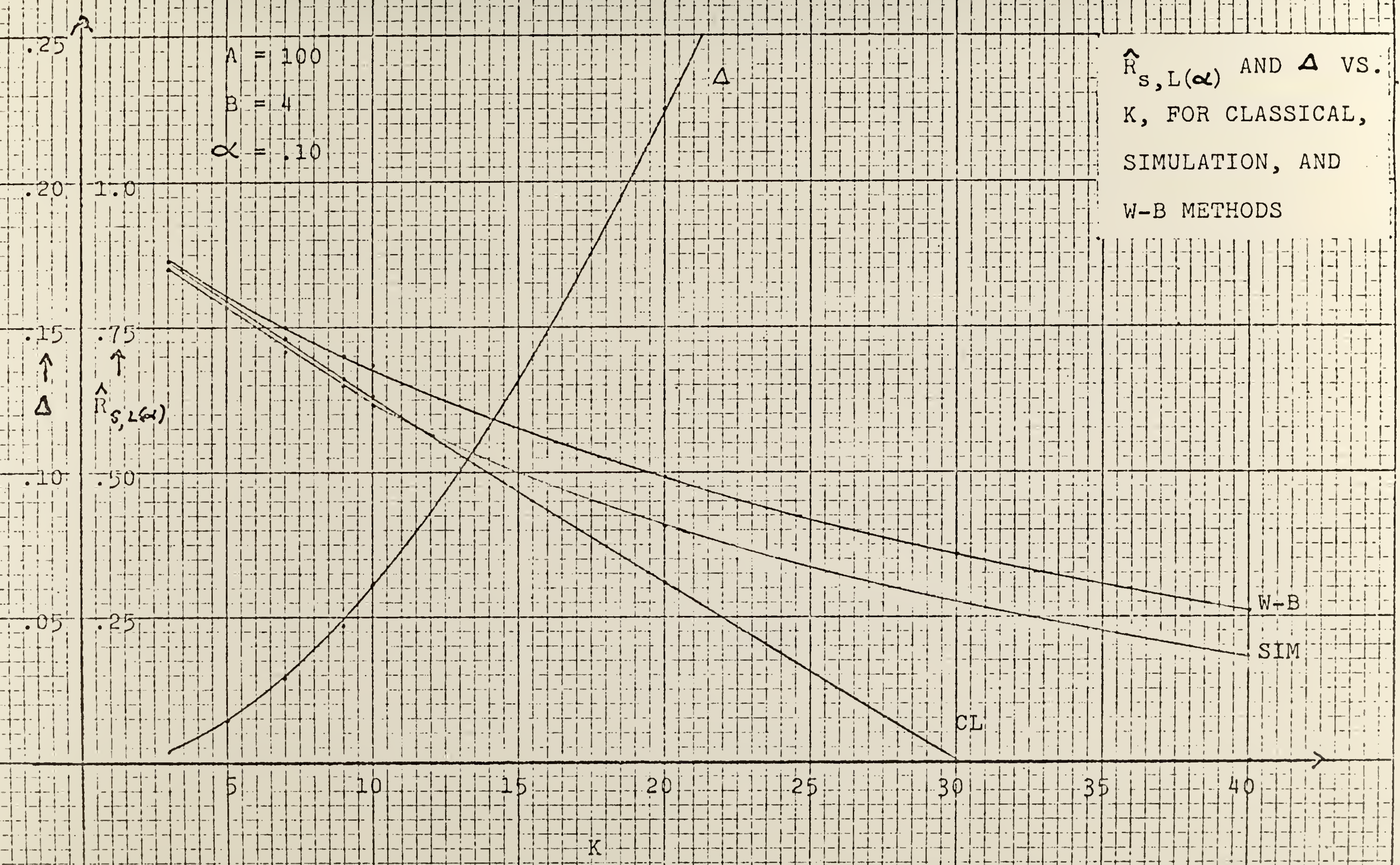

$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS
A = 50
B = 4
α = .10
Δ
$\hat{R}_{s,L(\alpha)}$
CL
W-B
SIM
K
.25
.20
.15
.10
.05
1.0
.75
.50
.25
5
10
15
20
25
30
35
40



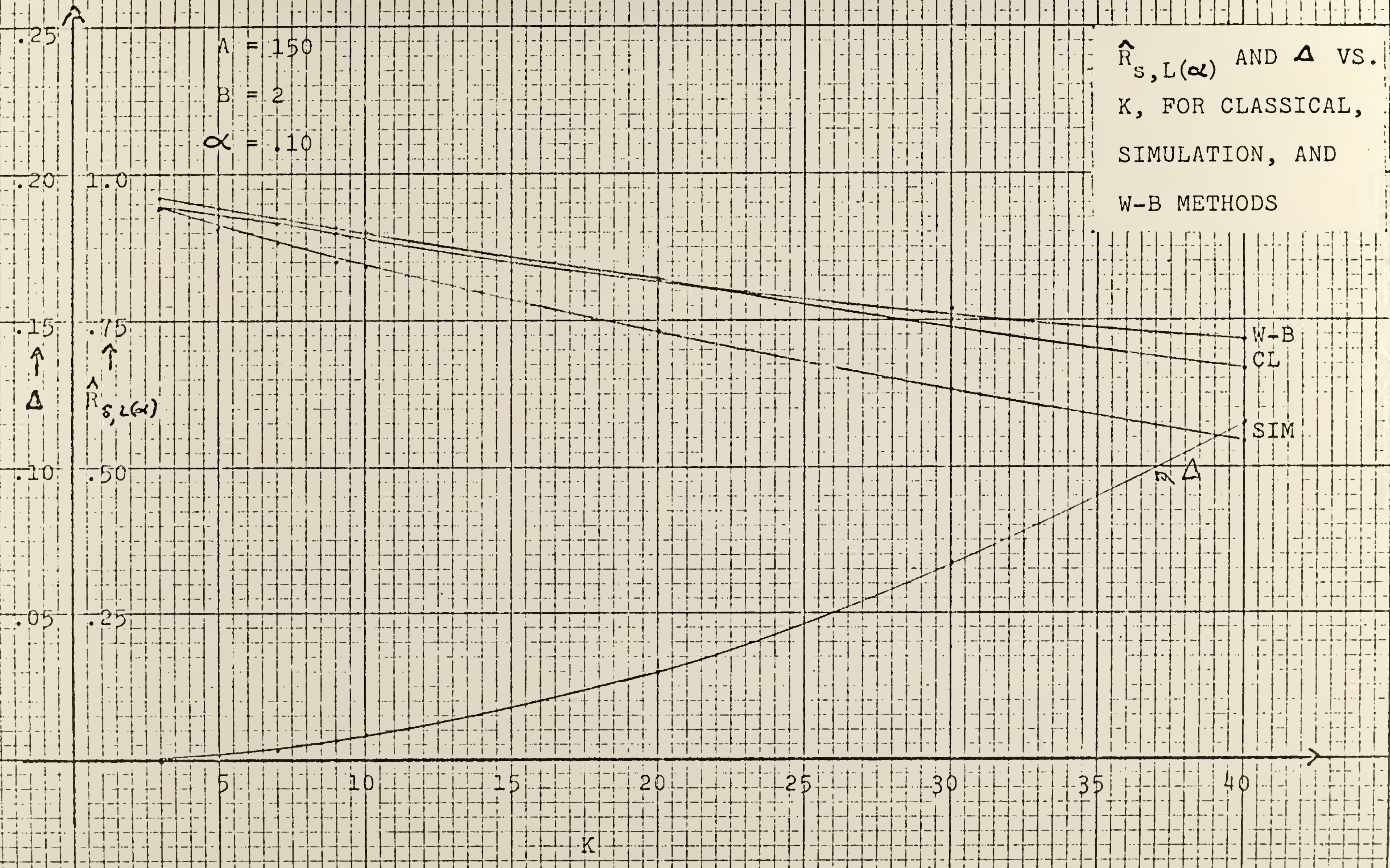
$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS

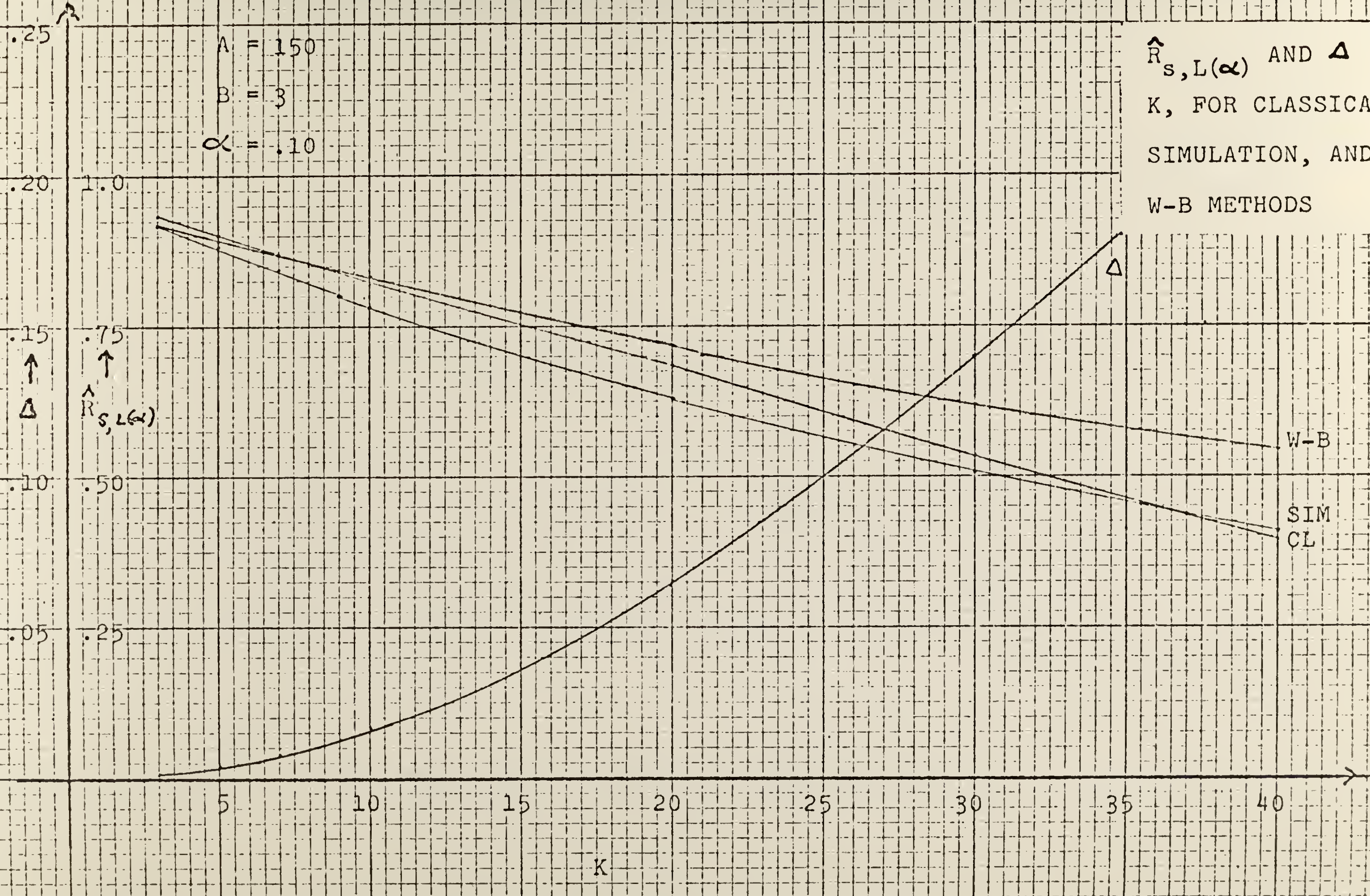


$\hat{R}_{s,L(\alpha)}$ AND Δ VS.
K, FOR CLASSICAL,
SIMULATION, AND
W-B METHODS
A = 100
B = 3
α = .10
.25
.20
.15
.10
.05
Δ
1.0
.75
.50
.25
$\hat{R}_{s,L(\alpha)}$
5
10
15
20
25
30
35
40
K
W-B
SIM
CL

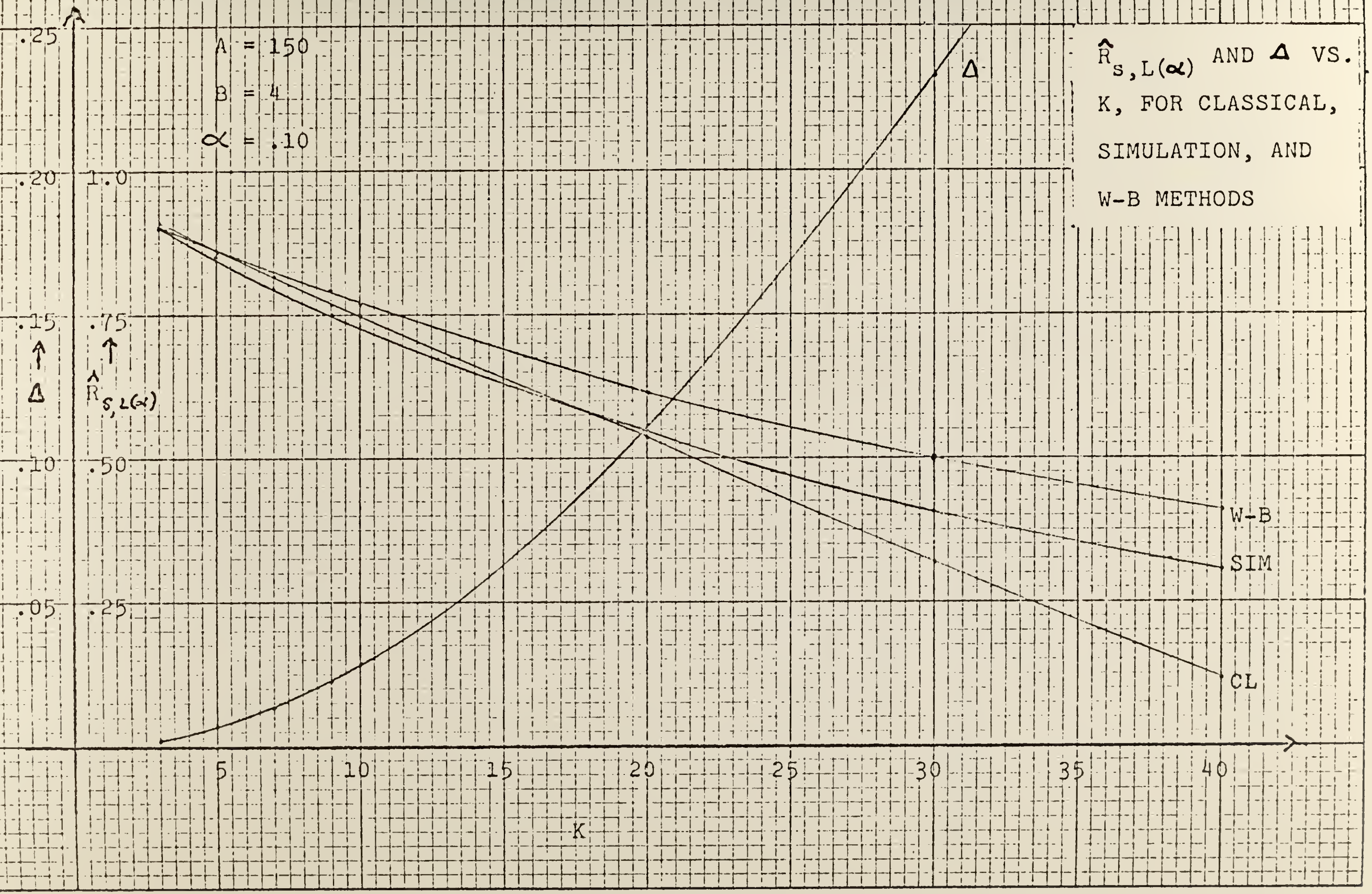


$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS


$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS
A = 150
B = 2
α = .10
.25
.20
.15
.10
.05
Δ
1.0
.75
.50
.25
$\hat{R}_{s,L(\alpha)}$
W-B
CL
SIM
Δ
5
10
15
20
25
30
35
40
K



$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS


$\hat{R}_{s,L(\alpha)}$ AND Δ VS. K, FOR CLASSICAL, SIMULATION, AND W-B METHODS
A = 150
B = 4
α = .10
Δ
W-B
SIM
CL
.25
.20
.15
.10
.05
1.0
.75
.50
.25
$\uparrow$ Δ
$\uparrow$ $\hat{R}_{s,L(\alpha)}$
5
10
15
20
25
30
35
40
K

DEFINITION OF VARIABLES IN COMPUTER PROGRAM

VARIABLE	
AA	Posterior parameter A
BB	Posterior parameter B
DATA(J)	Reliability of component (J)
DEV	Standard deviation of system reliability
IX	Random number seed
K	Number of components in series
SYSREL(I)	System reliability for replication (I)
XBAR	Mean value of system reliability

```
C
C      SIMULATION OF SERIAL SYSTEM RELIABILITY USING
C
C      BETA DISTRIBUTED COMPONENT RELIABILITIES
C
       DIMENSION SYSREL(1000)
       DIMENSION DATA(50)
       DIMENSION BB(50)
       DIMENSION AA(50)
       DATA AA/50*100./
       DATA BB/50*4./
       K=30
       NB=1000
       AB=NB
       IX=232
       SUM=0.0
       SUMSQ=0.0
       DO 66 I=1,NB
       SYSREL(I)=1.
       DO 67 J=1,K
       A=AA(J)
       B=BB(J)
C
C      GENERATE BETA DISTRIBUTED RANDOM VARIABLES
C
       CALL BETA(IX,IY,A,B,BTA)
       IX=IY
C
C      STORE VALUES OF RANDOM VARIABLES
C
       DATA(J)=BTA
C
C      COMPUTE SYSTEM RELIABILITY
C
       SYSREL(I)=SYSREL(I)*DATA(J)
   67  CONTINUE
       SUM=SUM+SYSREL(I)
       SUMSQ=SUMSQ+SYSREL(I)**2
   66  CONTINUE
       XBAR=SUM/AB
       VAR=(SUMSQ-AB*(XBAR**2))/(AB-1.)
       DEV=SQRT(VAR)
C
C      ORDER THE REPLICATED VALUES
C
       CALL SHELL (SYSREL,NB)
C
C      PRINT OUTPUT
C
       WRITE(6,22)K
       WRITE(6,13)XBAR,VAR,DEV
       SUMB=0.0
       DO 68 L=1,K
       SUMB=SUMB+BB(L)
   68  CONTINUE
       WRITE(6,14)SUMB
       WRITE(6,23)SYSREL(50),SYSREL(100),SYSREL(200)
   14  FORMAT(//,T5,'SUM OF B PARAMETERS',T25,F8.2)
   22  FORMAT('1',T5,'NO. OF COMPONENTS=',T25,I3)
   23  FORMAT(//,T5,'.05,.10, .20 PERCENTILE SIMULATION REL.'
      1,//,T20,3F7.3)
   13  FORMAT(////,T5,'OBSERVED MEAN',T20,F12.7,//,T5,
      1'OBSERVED VAR.',T20,F12.7,//,T5,'OBSERVED DEV.',
      2T20,F12.7)
       STOP
       END
```

```
      SUBROUTINE BETA(IX,IY,A,B,BTA)
      NN=B
      SUM=0.0
      DO 100 II=1,NN
      U=1./A
      IX=IX*65539
      R=J.5+IX*.2328306E-9
      IY=IX
      X=-(ALOG(R))*J
      SUM=SUM+X
      A=A+1.
  100 CONTINUE
      BTA=1./EXP(SUM)
      RETJRN
      END

      SUBROUTINE SHELL(X,N)
      DIMENSION X(N)
      M=N
   10 M=M/2
      IF(M.EQ.0) RETURN
      K=N-M
      J=1
   20 I=J
   30 IM=I+M
      IF(X(I).LE.X(IM)) GO TO 40
      S=X(I)
      X(I)=X(IM)
      X(IM)=S
      I=I-M
      IF(I.GE.1) GO TO 30
   40 J=J+1
      IF (J .LE. K) GO TO 20
      GO TO 10
      END
```

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13. ABSTRACT

This thesis examines three methods for calculating the 100(1-α)% lower confidence limits for the reliability of a K-sized series system. Assuming that each component reliability has a Beta distribution, identical posterior parameters A and B are assigned for each component. A computer simulation model is then developed to determine values of $\hat{R}_{s, L(\alpha)}$. The posterior parameters are then converted to attributes data, and $\hat{R}_{s, L(\alpha)}$ is computed using classical CHI-SQUARE methods, and the WOODS-BORSTING Method. The three values are then compared. Although no alternative approximations are examined, the results indicate that a high degree of accuracy in computing $\hat{R}_{s, L(\alpha)}$ is possible with the classical or WOODS-BORSTING methods, and that it may not be necessary to resort to costly simulation techniques to obtain values of $\hat{R}_{s, L(\alpha)}$.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Bayesian Reliability						
Reliability Confidence Limits						
Confidence Limits, Reliability						
Series System Reliability						

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